a.

Let f(n) = 10n^4 + 7n^2 + 3n

Let g(n) = n^4

By definition of tightly bound asymptotic complexity:

C1.g(n) <= f(n) <= C2.g(n), for all n>= n0, c > 0, n>= 1

C1.(n^4) <= 10n^4 + 7n^2 + 3n <= C2.(n^4)

Taking the left side:

C1.(n^4) <= 10n^4 + 7n^2 + 3n

Dividing through by n^4:

C1 <= 10 + 7/n^2 + 3/n^3

Let n = n0 = 1

C1 <= 10 + 7 + 3

C1 <= 20

Therefore, C1 = 20, n0 = 1

Taking the right side:

10n^4 + 7n^2 + 3n <= C2.(n^4)

Dividing through by n^4:

10 + 7/n^2 + 3/n^3 <= C2

Let n = n0 = 1

C2 >= 10 + 7 + 3

C2 >= 20

Therefore, C2 = 20, n0 = 1

Since, 20n^4 <= 10n^4 + 7n^2 + 3n <= 20.n^4, (g(n)) = n^4

b.

Let f(n) = 2n log n + 10n

Let g(n) = n log n

By definition of tightly bound asymptotic complexity:

C1.g(n) <= f(n) <= C2.g(n), for all n>= n0, c > 0, n>= 1

C1.(n log n) <= 2nlogn + 10 <= C2.(n log n)

Taking the left side:

C1.(n log n) <= 2nlogn + 10

Dividing through by n log n:

C1 <= 2 + 10/log n

Let n = n0 = 2

C1 <= 2 + 10/1 <= 2 + 10

C1 <= 12

Therefore, C1 = 12, n0 = 2

Taking the right side:

2nlogn + 10n <= C2.(n log n)

Dividing through by n log n:

C2 >= 2 + 10/log n

Let n = n0 = 2

C2 >= 2 + 10/1 <= 2 + 10

C2 >= 12

Therefore, C2 = 12, n0 = 2

Since, 12n log n <= 2nlogn + 10n <= 12n log n, theta(g(n)) = n log n